(Pages : 4) **4644**

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, September 2014
(2008 Scheme)
(Special Supplementary)
08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks.

1. The following is the p.d.f. of a random variable X :

x :	0	1	3	7	13
$P(X = x)$:	$\frac{1}{8}$	α	$\frac{1}{6}$	$\frac{1}{4}$	β

Find α and β such that $P(X^2 = 4X - 3) = \frac{1}{2}$.

2. A secretary makes two errors per page, on average. What is the probability that on the next page he or she will make
   i) 4 or more errors ?
   ii) no errors ?

3. Find the mean and variance of the uniform distribution.

4. If $f(x) = c\, e^{-x^2/2}, -\infty < X < \infty$ is the p.d.f. of a continuous random variable X, find $P(X > 1.75)$.

5. Using the principle of least squares, fit a straight line for the following data :

x :	1	5	10	15	20
y :	8	12	16	20	26

6. The following data were available $\overline{x} = 970, \overline{y} = 18, \sigma_x = 38, \sigma_y = 2$ and correlation coefficient $r = 0.6$. Find the value of X when Y = 20.

P.T.O.

7. A random sample of size 10 from a normal population with S.D 5 gave the following observations 65, 72, 71, 85, 73, 76, 67, 70, 74, 76. Calculate the 95% confidence interval for the population mean.

8. Show that, if a wide sense stationary process X(t) is periodic in time, then the autocorrelation $R_X(\tau)$ is periodic in τ.

9. X(t) is a wide sense stationary process and Y(t) = X(t – a) is a delayed version of X(t), where a is a constant. Show that the power spectral densities of X(t) and Y(t) are equal.

10. Obtain the probability distribution of the time between two consecutive occurrences of a Poisson process.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that
    i) at least 10 survive
    ii) from 3 to 8 survive, and
    iii) exactly 5 survive ?

    b) In a test on 2000 electric bulbs, it was found that the life of a particular was normally distributed with an average of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for
    i) more than 2150 hours
    ii) less than 1950 hours and
    iii) more than 1920 hours but less than 2160 hours.

    c) The time that a machine will run without repair is exponentially distributed with mean 150 days. Find the probability that such a machine will (i) have to repair in less than 100 days, (ii) not have to repair in atleast 175 days.

12. a) A continuous random variable X has the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \le 1 \\ c(x-1)^4 & \text{if } 1 < x \le 3 \\ 1 & \text{for } x \ge 3 \end{cases}$$

Find c and P(2 < X < 4).

b) The marks obtained by a batch of students in Mathematics are approximately normally distributed with mean 65 and standard deviation 5. If 5 students are selected at random from this group, what is the probability that atleast one of them will score above 75 ?

c) Subway trains on a certain line run every half hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait (i) at least twenty minutes and (ii) atmost 10 minutes ?

Module – II

13. a) Calculate the correlation coefficient for the following data :

x :	75	30	60	80	53	35	15	40
y :	85	45	54	91	58	63	35	43

b) A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Would you agree with this claim if random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use 1% level of significance.

c) A die is thrown 9000 times and a throw of 1 or 3 was observed 3240 times. Show that the die can not be regarded as an unbiased one.

14. a) Using the principle of least squares, fit a parabola of the form $y = a + bx + cx^2$ for the following data :

X :	0	1	2	3	4	5
Y :	14	18	22	27	38	40

b) At a highway location with poor geometrics, the mean spot speed observed with a sample of 200 vehicles was 58.3 km/hour with a SD of 12.2 km/hour. After effecting improvements to the geometrics, the mean speed observed with a sample of 250 vehicles was 61.2 km/hour with a SD of 9.8 km/hour. Has there been a significant increase in the speed after the improvements ?

c) A certain injection administered to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the injuction, in general, accompanied by an increase in blood pressure. (Use 5% level of significance.)

Module – III

15. a) A continuous two-dimensional random variable (X, Y) has joint density given by

$$f(x, y) = \begin{cases} c & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

i) the value of c and ii) the marginal distributions iii) Are X and Y independent ? Justify.

b) Show that the random process $X(t) = A\cos(\omega t) + B\sin(\omega t)$ where ω is a constant, A and B are independent random variables with zero mean and equal variance, is wide sense stationary.

c) The power spectral density of a wide sense stationary process X(t) is

$$S_X(\omega) = \begin{cases} 1+\omega^2, & |\omega| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the power contained in the frequency band $-0.5 \le \omega \le 0.5$?

16. a) The transition probability matrix of a Markov Chain with two states 1 and 2 is

$\begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{pmatrix}$ and the initial probability distribution is

$P(X_0 = 1) = 0.3$, $P(X_0 = 2) = 0.7$. Find

i) $P(X_2 = 2 \mid X_0 = 2)$

ii) $P(X_3 = 1)$

iii) $P(X_3 = 2, X_2 = 1, X_1 = 2, X_0 = 1)$

iv) the steady state distribution.

b) A radio active source emits particles at the rate of 4 per minute in accordance with a Poisson process. Each emitted particle has probability 1/3 of being recorded by a device.

i) Find the probability that during each of two consecutive minutes, at least 3 particles are emitted.

ii) Find the probability that at least 4 particles are recorded in a 3 minute period.

iii) What is the probability that the time between two emissions is at least 1 minute ?

iv) What is the probability that the time between two recordings is at least 1 minute ?
